

EXPLICIT VG GUIDANCE ALGORITHM FOR A SOLID POWERED CLOSED LOOP GUIDANCE MISSION

S. Dasgupta and S. Ramakrishna

Control and Guidance Division, Vikram Sarabhai Space Centre, Trivandrum, India

Abstract. An explicit form of velocity to be gained guidance algorithm is proposed for ASLV mission of Indian Space Research Organisation. ASLV mission is to inject a 150 kg payload into a 400 Km near circular orbit using a four stage solid propellant system. Since thrust termination cannot be commanded when the lower stages overperform during the flight, guidance algorithm commands the final stage to be reoriented, such that the final orbit is close to circular. When underperformance occurs, a velocity augmentation system is activated to pull up the performance to near nominal. It is shown through simulation that for 1 sigma over and under performance, the orbit eccentricity is less than 0.00055.

Key words: Guidance systems; closed loop systems; space vehicles; target tracking; required velocity.

NOMENCLATURE

$x_I \ y_I \ z_I$	Topo centric Inertial coordinates parallel to LPI, with origin at Earth Centre.
$v_1 \ v_2$	Required velocity in guidance co-ordinates.
μ	Gravitational constant.
r	Distance of vehicle CG from earth centre
H	desired angular momentum during long coast
A	desired apogee at end of long coast.
v	local horizontal component of velocity required.
ℓ	direction cosine of normal to the current orbit plane.
\bar{R}	position vector in LPI
\bar{V}	velocity vector in LPI
θ	angle between desired orbit plane and current orbit plane.
k, k_1	Constants
ω_c	steering command rate
\bar{v}_G	velocity to be gained vector
\bar{a}	acceleration vector
θ_c	pitch attitude command
ψ_c	yaw attitude command
$v_{GXN}, v_{GYN}, v_{GZN}$	normalised velocity to be gained components
(v_3, β)	velocity and flight path angle at any time during long coast.
v_4	equivalent velocity increment due to fourth stage, when added impulsively.
α	angle, from local horizontal, at which fourth stage is ignited
Sv_3	velocity decrement due to gravity over the duration of fourth stage burning, acting along local vertical.
(v_{gr}, β_{gr})	vector sum of (v_3, β) and $(Sv_3, -180^\circ)$
(v_n, β_n)	Resultant velocity vector sum of (v_{gr}, β_{gr}) and (v_4, α_{fr})

INTRODUCTION

Multi stage launch vehicles are steered in flight in order to reach desired altitude, velocity, flight path angle and flight azimuth at the end of thrust, so that the payload is inserted into the desired orbit. While payload optimisation and mission profile are generated assuming a nominal vehicle performance, the inflight performance of the vehicle's propulsion system and disturbances due to aerodynamics can cause significant dispersions in the orbit attained. This calls for a closed loop guided system that can measure position, velocity and attitude of the vehicle at any time and determine from them, the steering profile to be followed in order to achieve the desired orbit. Closed loop guidance algorithm is invoked to derive the steering commands in pitch, yaw and roll motions.

Indian Space Research Organisation plans to have its first closed loop guided mission with the launch vehicle ASLV. This is designed to put 150 Kg. payload into a near earth circular orbit with specified tolerances in orbit eccentricity and inclination. ASLV is an all solid powered launch vehicle and has no provision for thrust termination.

During the atmospheric phase, the vehicle executes a predetermined pitching sequence. Closed loop guidance is initiated at second stage, and continues until the end of third stage burnout. This is followed by a long coast during which the vehicle gain altitude. At the apogee of the coast phase the vehicle is spun, fourth stage separated and ignited to impart the necessary velocity increment to the payload.

Several guidance algorithms are reported in literature (Tounsend; Battin 1982 Gage 1968). Broadly, these can be categorised into explicit and implicit schemes. In explicit guidance, closed form analytical solution is obtained for a simplified TPBV problem. Several assumptions, such as those of uniform gravity, constant thrust and propellant massflow rate are made in order to arrive at analytical solutions. In implicit schemes, required velocities are obtained from a nominal profile and are stored as polynomials of perturbation in position, time, velocity etc. about the nominal burnout state. Generally, explicit schemes require higher computation time, while implicit schemes require a larger memory.

GUIDANCE ALGORITHM FOR ASLV

In this paper, a guidance algorithm is proposed, that is an implicit form of classical velocity to be gained (VG) technique. While steering is achieved using cross product steering command of VG guidance, the required velocity is computed explicitly, using closed form analytical solutions. The target is specified as a desired coast apogee, that must be reached with a specified angular momentum. Using classical keplerian laws, the velocity required to attain the specified terminal conditions from any altitude, are analytically evaluated. This obviates the need for storing predetermined nominal profile, of required velocities, either as

tables or as polynomials, as is generally adopted in classical VG techniques. Since closed form Keplerian solution is used to predict the path connecting the current position of vehicle to the desired terminal state, the TPBV problem formulation is not necessary. Consequently, the memory requirement as well as computational load are expected to be less than conventional implicit and explicit schemes, respectively.

In ASLV the fourth stage is a solid motor that is spin stabilised prior to ignition. There is no provision for thrust termination in any of the stages of ASLV. Consequently, when any or all of the stages overperform, i.e. impart higher velocity than is nominally expected, the resultant orbit will be quite eccentric. In order to achieve a near circular orbit, prior to ignition the final stage is reoriented from the local horizontal attitude by an angle determined on-board. This angle is evaluated as a function of overperformance by lower stages.

In case the lower stages under perform, a velocity Augmented system (VAS) mounted axially on ASLV, is fired during the coast phase. The orientation of the vehicle and the duration of VAS firing are determined on-board, depending on the level of under performance.

The capacity of VAS, to impart velocity is limited to 25 m/s, study indicates that under dispersions, it may be necessary to augment velocity by as much as 53 m/s. Left uncorrected, this will again result in an eccentric orbit. Effect of removing VAS (that weighs nearly 42 Kg) from ASLV and depending solely on the reorientation algorithm to achieve low eccentricity, is studied and results are presented in this paper.

Coordinates System

Guidance coordinates ($X_G Y_G Z_G$) are illustrated in Fig.1. Inertial coordinates ($X_I Y_I Z_I$) are Topocentric and defined along the launch point vertical, down range and normal to down range respectively.

Required Velocity

Target conditions are the states attained by nominal ASLV vehicle at the apogee of the third stage coast and are obtained from a nominal, optimised open loop trajectory. Using Keplerian equations to describe nature of a body under Central force field, for any given altitude, the velocity required to attain the specified altitude and velocity at cost apogee are given in guidance coordinates as

$$u^2 = \frac{\mu}{r} \left[2 + r \left(\frac{H^2}{\mu} - 2A \right) / A^2 \right] - H^2 / r^2$$

$$v = H/r$$

Angle-to-go

The plane of orbit, if coast were to commence at the current state, is given by $\vec{L} = \vec{R} \times \vec{V}$. The angle between this plane and the desired orbit plane is defined as the Angle-to-gos. Velocity component V_1 , normal to the orbit plant, is then, where K_1 is a constant, $V_1 = K_1 \cdot \delta$

Since navigation is performed in LPI coordinates (u, v_1, v_2) are transformed to LPI and compared with current velocities in LPI, to give the velocity to be gained vector in LPI coordinate system.

Crossproduct Steering

Battin (1982) has shown that if the vehicle is steered according to

$$\omega_c = K(\bar{V}_G \times \dot{V}_G) \quad (1)$$

where ω_c is the commanded rate,

\bar{V}_G is the velocity to be gained

\dot{V}_G is the derivative of \bar{V}_G

the path followed by the vehicle is fuel optimum.

An alternate scheme is to use \bar{a} , the acceleration vector, instead of \dot{V}_G so that

$$\omega_c = K(\bar{V}_G \times \bar{a}) \quad (2)$$

Thus, by computing \bar{V}_G analytically and steering through crossproduct, pitch and yaw commands are generated with small computational load, small data storage and being an explicit algorithm, gives accurate injection conditions.

The flow chart for steering command generation during thrust phase is shown in fig.2. It may be observed from Fig.2 that required velocity is initially computed in guidance coordinates in terms of 'in-plane' and 'out-of-plane' velocity components. These are transformed to inertial co-ordinate systems for calculating velocity to be gained in inertial system. Steering rate is then computed through (2). Near the end of guided phase, steering angle is computed according to

$$\theta_c = \tan^{-1}(V_{GZN}/V_{GXN}), \quad (3)$$

$$\psi_c = -\sin^{-1}[V_{GYN}] \quad (4)$$

Details of mathematical formulation are discussed by Dasgupta and Ramakrishna (1982)

Closed loop Guidance in Coast Phase

In case the lower stages under perform, the apogee and angular momentum at apogee are enhanced by VAS. At the beginning of coast, whenever the predicted apogee is in error greater than a preset amount, VAS is fired. During this period of VAS actuation, the vehicle is oriented according to equations (3) and (4). It is also ensured that the vehicle tracking error is within the stipulated control system dead-zone, prior to VAS actuation. When the vehicle's velocity vector is nearly locally horizontal, VAS actuation is based on predicted velocity errors rather than apogee. This phase is termed 'fine trim' of VAS and velocity error of less than 0.1 m/s and 50m can be achieved at the end of VAS actuation. During this phase, the vehicle is oriented to the locally horizontal. Since the vehicles flight path angle is nearly 90°(measured from

local vertical) altitude of ASLV is not significantly affected by the 'fine trim' phase of VAS firing.

The 3 σ under performance level of ASLV lower stages is equivalent of 53 m/s at third stage burnout. However, VAS tank capacity is limited to a velocity augmentation of only 25 m/s. As may be expected, for underperformances demanding velocity corrections beyond 25 m/s, the eccentricity of the resultant orbit is high, and beyond the specified 0.0025.

In order to hold eccentricity to within specifications, under $\pm 3\sigma$ levels of vehicle dispersions, VAS, which weighs 42 Kg, has been removed from the structural weight of third stage, in the simulation. This leaves a level of overperformance of nearly 70 m/s. at third stage burn out. Since, the $\pm 3\sigma$ dispersion is 53 m/s, sufficient margin is available in the vehicle to handle the expected dispersions, and velocity augmentation is not called for.

In case the lower stages overperform, the apogee of the coast phase will be higher than that desired. If the final stage is fired at this apogee, the resultant orbit may be highly eccentric. Fig.3 gives the velocity when the final stage is ignited at an angle α to the local horizontal. In actual practice, the velocity addition during the stage will be along an arc subtending an angle ξ at earth centre. However, for simplicity an impulsive addition of velocity is assumed during fourth stage. V_4 is the effective impulsive velocity increment, the value being determined from simulation of a nominal fourth stage trajectory.

By forcing the following conditions:

Magnitude of \bar{V}_R equal to the predicted circular velocity at end of final stage, and \bar{V}_R locally horizontal at burn out of final stage, where \bar{V}_R is the predicted velocity vector at fourth stage burnout.

the reorientation angle α and instant of final stage ignition is determined, such that the resultant orbit is nearly circular. The final stage is reoriented and spun before ignition. In fig.3, δv_g is the decrement in velocity due to acceleration over the duration of final stage thrust. \bar{V}_{gr} is the resultant of (V_5, β) and $(\delta v_g, \eta)V_4$ is estimated as

$$V_4 = \frac{\text{Thrust}}{\text{Mass}} dt, \quad \text{where } t_f \text{ is the burning time of stage.}$$

At any instant, let \bar{R} be the position of the vehicle. Circular velocity V_c at this altitude is $V_c = \sqrt{\mu/r}$ where μ is the gravitational constant.

When \bar{V}_4 is added to \bar{V}_3 impulsively at an angle α to local horizontal, such that

$$\alpha = \beta_{gr} - \sin^{-1}[(V_c^2 - V_{gr}^2 - V_4^2)/2 V_{gr} V_4] \quad (5)$$

the resultant velocity is equal in magnitude to V_c .

The flight path angle β_r is given by

$$\beta_r = \pi/2 + (\alpha - \gamma)$$

where $\tan \gamma = (V_{gr} \cos(\beta_{gr} - \alpha)) / V_{gr} + V_{gr} \sin(\beta_{gr} - \alpha)$

$$(7)$$

In flight, the velocity addition is not impulsive but over the duration of fourth stage burning. It is estimated that the local vertical direction changes by where δ_x is nearly 2.3 deg over this duration. Thus in order to attain a flight path angle of 90° at fourth stage burn out,

$$\alpha - \gamma = \delta_x \quad (8)$$

During the coast phase, the vehicle pitch is continually commanded to the local horizontal. Equation (5) and (7) are solved for α and γ respectively. δ_x is evaluated at third stage, burnout as a function of residual velocity to be gained, V_{gbo}

$$\delta_x = \delta_{x_0} + k_1, V_{gbo} \quad (9)$$

k_1 are precomputed through simulation, δ_{x_0} being 2.3 degrees for ASLV.

During coast phase, condition stipulated in equation (8) is checked at each guidance cycle, and when satisfied, the vehicle is reoriented through α and coast phase is terminated, followed by spin up, separation and final stage ignition. Thus, it may be noted, that no prediction is made on the time at which fourth stage is to be ignited.

SIMULATION RESULTS

ASLV is designed to put 150 Kg payload in near 400 km, near circular orbit, eccentricity being less than .0025. The target for guidance algorithm is drawn from nominal fourth stage ignition states and are

Altitude	=400.644 Kms.
Velocity	=5198.1165 m/s.
direction cosines of Normal to de- sired orbit plane.	=-0.0578, 0.99757, 0.038696

These are to be achieved at end of 3rd stage long coast.

Table 2 gives the coast end condition reached through simulation, under various perturbations in thrust, weight and other parameters. RSS estimate of velocity error at 3rd stage burnout is 17.0 m/s. (1 sigma). For underperformances, VAS logic activates the axial thrusters, VAS trim is called when flight path angle is 88°. Predicted velocity error at the end of fine trim firing is 0.1 m/s. Altitude error is less than 50.0 m.

The effect of out of plane disturbances due to thrust misalignment and winds is shown in Table 3. Under worst case of disturbances, the error in inclination is 0.037°.

When vehicle overperforms, the final stage is reoriented through an angle α depending on the level of overperformances. For a nominal final stage performance, the orbit reached for various levels of overperformances in lower stages is given at 1 sigma dispersion in parameters, orbit eccentricity is 0.00054. When VAS capacity is limited to 25 m/s,

eccentricity of orbit for underperformances larger than 25 m/s, is beyond the specification of 0.0025. e.g. for underperformance of 53 m/s, eccentricity is 0.00749. Table 5 gives the orbital dispersions for $\pm 3\sigma$ dispersions, when VAS weight is removed from the third stage for the nominal vehicle. $V_{gbo}=-71.2$ m/s $\pm 3\sigma$ dispersions in vehicle parameters correspond to a V_{gbo} of -122.9 m/s and -19.2 m/s respectively. With reorientation angle α indicated in Table 5, eccentricities are kept within the specification. Analytical expressions for sensitivity of A_p to errors in u, v are derived by Dasgupta (3). For ASLV, at third stage burn out, $\partial A_p / \partial u = 93.3$ m/m/s, $\partial A_p / \partial v = 74.8$ m/m/s. Also $\partial u / \partial r = -.0033$ m/s/m and $\partial v / \partial r = -.00082$ m/s/m. Thus $\partial A_p / \partial r = -.369226$

The navigation system of ASLV has an error bound of 300m at third stage burnout. Hence, the apogee reached at end of long coast is in error by a maximum of 110.76 m from the desired value.

CONCLUSIONS

This paper discusses the guidance strategy that is followed for guiding an all-solid-propellant launch vehicle ASLV into a desired orbit. In addition to a steering during thrust phase, coast phase guidance with axial thruster firing and final stage reorientation is proposed as the strategy to overcome the unpredictable effects of under and overperformance of launch vehicle. Simulation results illustrate the performance of the algorithm.

REFERENCES

Battin, R.H. (1982) Space Guidance Evolution - A personal approach. Journal & Guidance, Control and Dynamics March - April 1982, Vol.5, 97-110

Gage, M.W. (1968) FE Guidance and its application to ELDO Lunacher Vehicle, RAE 68201.

Das Gupta, S., and S. Ramakrishna (1982) An improved algorithm for terminal VG Guidance and performance studies of ASLV mission. VSSC:TR:032:82

Townsend, G.E. Abbott, A.S. and Palmer, RR Guidance Flight Mechanics and trajectory optimisation. Vol VIII: Boost Guidance Equations NASA:CR:1007.

TABLE 1 3 sigma PERTURBATION LEVELS

S1. Stage No:	Parameter	Disper-	VG at 3rd stage burn-	out (m/s)
1	Strapons	Thrust	+1.0%	-17.02
2	Strapons	Thrust	-1.0%	+13.86
3	Core	Thrust	+1.0%	-19.34
4	Core	Thrust	-1.0%	+16.09
5	Second	Thrust	+1.0%	-19.66
6	Second	Thrust	-1.0%	+16.39
7	Third	Thrust	+1.0%	-20.46
8	Third	Thrust	-1.0%	+17.18
9	Strapons	Str.wt.	-20.0Kgs.	- 3.40
10	Strapons	Str.wt.	+20.0 "	- 0.06
11	Core	Str.wt.	-20 "	-5.32
12	Core	Str.wt.	+20 "	+1.97
13	Second	Str.wt.	-15 "	-7.56
14	Second	Str.wt.	+15 "	+4.19
15	TMird	Str.wt.	-7.5 "	-10.98
16	Third	Str.wt.	+7.5 "	+ 7.57
17	Strapons	Prop.wt.	+30 "	-5.85
18	Strapons	Prop.wt.	-30 "	+2.54
19	Core	Prop.wt.	+30 "	-4.98
20	Core	Prop.wt.	-30 "	+1.65
21	Second	Prop.wt.	+20 "	-10.10
22	Second	Prop.wt.	-20 "	+ 6.81
23	Third	Prop.wt.	+7.5 "	-11.10
24	Third	Prop.wt.	-7.5 "	+ 7.81
25	First	Drag.coeff	-15.0%	-30.06
26	First	Drag.coeff	+15 "	+26.53

1	2	3	4	5
13	Second	Str.wt.	+1558.83	+05.3435
14	Second	Str.wt.	-0774.53	-03.1565
15	Third	Str.wt.	+2285.80	+07.7135
16	Third	Str.wt.	-1389.77	-05.7265
17	Strapons	Prop.wt.	+1190.22	+04.1750
18	Strapons	Prop.wt.	-0460.39	-01.8871
19	Core	Prop.wt.	+1018.86	+03.5411
20	Core	Prop.wt.	-0325.87	-01.0220
21	Second	Prop.wt.	+1083.51	+07.1330
22	Second	Prop.wt.	-1254.32	-05.1426
23	TMird	Prop.wt.	+2287.59	+07.8473
24	Third	Prop.wt.	-1436.98	-05.9033
25	First	Drag.coeff	+6150.09	+21.1236
26	First	Drag.	" -4880.30	-20.0640

TABLE 3 EFFECT OF YAW DISTURBANCE ON ORBITAL ERRORS

Note: 1. Misalignments in thrust for first, second and third stages are 0.3, 0.28, 0.15 degrees respectively.

2. Misalignment in yaw plane is 90°

S. No:	Dist. occurs in	Yaw nation error	Inclination (deg)	Errors at injection (predicted)	Re-
				Apogee vel. at apogee (mtr)	marks
				(m/s)	
				(RT-AP) (VT-VAP)	

1.	Strapons stage	0.0003	71.196	0.018	RT, VT-target value AP,
2.	Core stage	0.021	829.319	3.185	VAP predicted values.
3.	Second stage	0.012	-32.043	-0.310	
4.	Third stage	0.002	-289.124	-1.305	
5.	All stages	0.037	1089.629	5.592	

TABLE 2 INJECTION ERRORS FOR 3 SIGMA PERTURBATIONS

S1. Stage	Para-	Injection errors without cut off	
		meter	Apogee(mtrs)
1	Strapons	Thrust	+0493.92
2	Strapons	Thrust	-2555.59
3	Core	Thrust	+3974.50
4	Core	Thrust	-2942.53
5	Second	Thrust	+4130.97
6	Second	Thrust	-2984.93
7	Third	Thrust	+4286.83
8	Third	Thrust	-3109.09
9	Strapons	Str.wt.	+0703.13
10	Strapons	Str.wt.	+0016.37
11	Core	Str.wt.	+1077.87
12	Core	Str.wt.	-037.93

TABLE 4 EFFECT OF OVERPERFORMANCE
ON ORBITAL ERRORS

Sl. No:	VG (m/s)	Degrees	Apogee Kms	Perigee Kms	Mean Kms.	Dispersion Kms.	Eccentricity
1	-10.8	6.930	403.423	410.090	402.250	+1.16	0.00017
2	-16.8	8.390	406.032	398.729	402.300	+3.60	0.00054
3	-26.2	10.166	408.454	394.727	401.590	+6.86	0.00101
4	-39.3	12.279	410.928	386.407	398.677	+12.26	0.00181
5	-52.5	14.135	407.726	378.383	393.083	+14.70	0.00217

TABLE 5
ORBIT WITH NO VAS ON ASLV

Vgbo m/s.	deg	Eccentricity
-19.2	9.7	0.0012
-38.6	12.3	0.00023
-53.12	14.11	0.00046
-71.2	15.9	0.00133
-89.21	18.2	0.00040
-103.5	19.0	0.00146
-122.9	20.3	0.00221

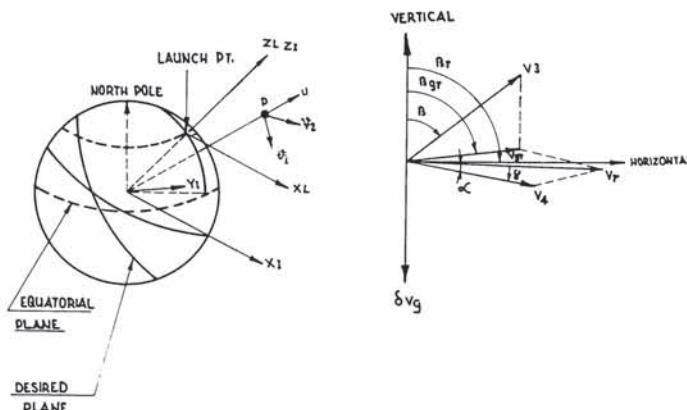


FIG.-1. CO ORDINATE SYSTEM.

FIG.-3. VELOCITY DIAGRAM FOR FOURTH STAGE.

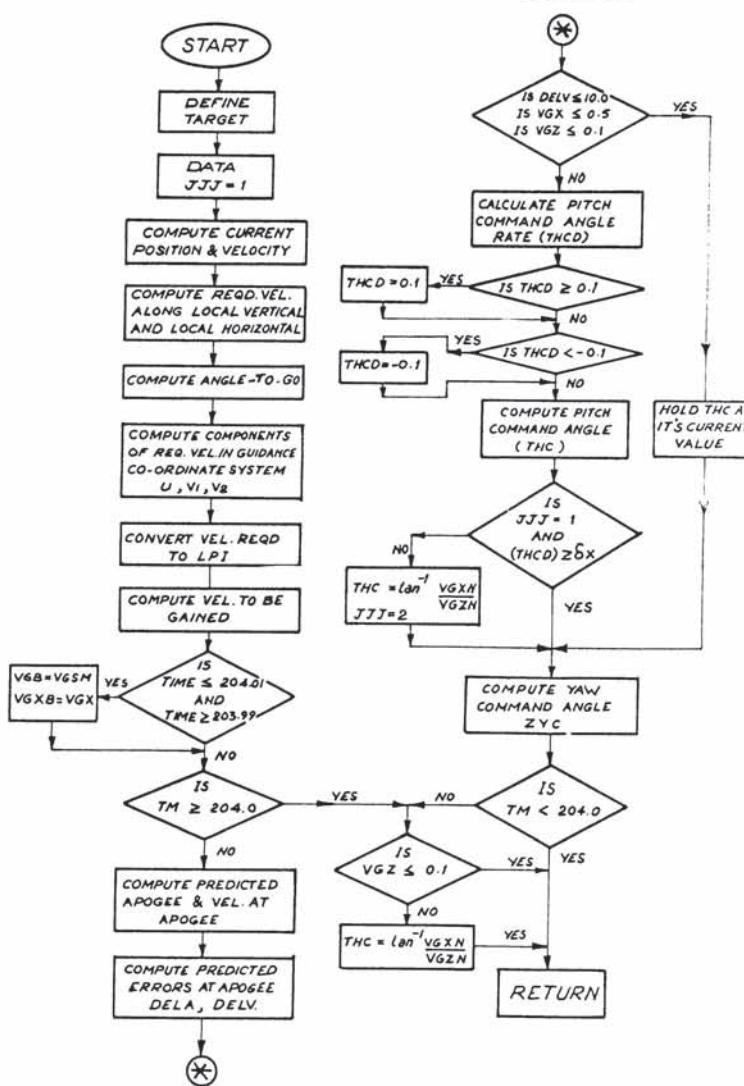


FIG. 2. BOOST PHASE GUIDANCE